B.Sc - Part III

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Example of a bounded function which is not R- integrable.

Let a function f be defined on $[a,b]$ as follows:

$$f(x) = \begin{cases} 1, \text{ when } x \text{ is rational} \\ 0 \text{ when } x \text{ is irrational} \end{cases}$$

Show that f is not integrable.

Soln Here the function $f : [a,b] \to R$ i.e. defined by $f(x) = 1$ if x is rational

$= 0$ if x is irrational.

Clearly f is a bounded function over $[a,b]$.

we shall show that f is not R-integrable over $[a,b]$

Let $P = \{a = x_0, x_1, x_2 \ldots x_n = b\}$ be any partition of $[a,b]$

Let m_r and M_r denote the glb and lub respectively of f in $[x_{r-1}, x_r]$ ~~from~~ and let m, M be the glb and lub of f in $[a,b]$.

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Then $m = m_r = 0$ and $M = M_r = 1$; $r = 1, 2 \ldots$

Since every subinterval $[x_{r-1}, x_r]$ of $[a, b]$ contains rational as well as irrational points.

Now $U(P) = \sum_{r=1}^{n} M_r \delta_r = 1 \sum_{r=1}^{n} \delta_r = 1$

$\because M_r = 1$ for each r $= 1(1-0) = 1$

$L(P) = \sum_{r=1}^{n} m_r \delta_r = 0 \sum_{r=1}^{n} \delta_r$;

$\because m_r = 0$ for each r

$= 0(1-0) = 0$

Thus $L(P) = 0$ and $U(P) = 1$ for every partition P of $[a, b]$

Hence $\text{lub}\{L(P)\} = 0$ i.e. $\underline{\int_a^b} f = 0$

Since $\underline{\int_a^b} f \neq \overline{\int_a^b} f$, therefore f is not R integrable over $[a, b]$

although it is bdd over $[a, b]$.